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MARKET MAKING AND REVERSAL ON THE STOCK EXCHANGE

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The accurate record of stock market ticker prices displays striking properties of dependence. We find for example that after a decline of $\frac{1}{8}$ of a point between transactions, an advance on the next transaction is three times as likely as a decline. Further examinations disclose that after two price changes in the same direction, the odds in favor of a continuation in that direction are almost twice as great as after two changes in opposite directions.

The dealer (specialist) in a stock typically quotes the market by announcing the highest buy order and lowest sell order carried on his book. But these orders tend to be concentrated at integers (26, 43), halves ($26\frac{1}{2}$, $43\frac{1}{2}$), quarters and odd eighths in descending preference. This non-uniform distribution of orders produces some non-random effects in stock price motion. These properties of the stock market are typical of markets in many second-hand goods.

1. INTRODUCTION

OUR objective in this report is to find laws of price fluctuation in the stock market. We shall examine the most elementary data discernible, the record of successive transactions on the ticker tape. This record, which is published in usable form by Francis Emory Fitch, Inc., provides precise and abundant information.

It is convenient at the outset to compare the movements of successive transactions with those predicted by a random walk model, the epitome of unrelieved bedlam. The proponents of the random walk state that changes in the price of consecutive transactions are distributed independently of each other. The assumption of independence means that the change in price following the current transaction will not be influenced by the sequence of preceding price changes. That is:

$$P(\Delta Y_t = X \mid \Delta Y_{t-1}, \Delta Y_{t-2}, \cdots) = P(\Delta Y_t = X),$$

where

$$\Delta Y_t = Y_{t+1} - Y_t \qquad (t = 1, 2, \cdots n) \tag{1}$$

and Y_t is the price at which the tth transaction occurred.[1] Although the probability of an advance in the future can be estimated from the relative frequency of advances in the past, this probability does not change from transaction to transaction.

Godfrey, Granger, and Morgenstein [5] have argued that model (1) provides a reasonably accurate description of market behavior. Other writers have stated that model (1) fits when Y_t represents the price at time t rather than the price at the tth transaction (cf. the articles in [2]). Finally, some scholars define a series as independent unless an investor can use the observed dependence to increase his expected profits [4].

In section 2, however, an analysis of a sample of Dow Jones Industrial Stocks shows considerable dependence between transactions. The results indicate that after a price rise the odds are approximately 3 to 1 that the next non-zero change will be a decline, but after a decline the odds are about 3 to 1 in favor of a rise. Therefore, another model may be more appropriate for the explanation of these changes. In section 3, we analyze the process of change in ticker prices by employing statistical techniques developed and recommended by Goodman [1, 6]. We find, for example, that after two changes in the same direction the odds in favor of a continuation in the direction of a particular price move are almost twice as great as after two changes in alternate directions.

With this empirical evidence on non-randomness in mind, we consider the structure of developed trading markets with particular applications to the stock market in section 4. This leads to definite predictions about the properties of stock prices. These predictions are tested in section 5 by a second sample of data taken from all the listed stocks. The predictions are in the main confirmed. They are natural consequences of the market making process.

2. REVERSALS IN DOW STOCKS

Our purpose here is to examine the correspondence between the movements of ticker prices and the predictions of the random walk hypothesis. The data consist of the complete set of ticker prices of six of the first seven stocks in the Dow Jones Industrial Averages for the twenty-two trading days of October, 1964. (See Table I.)[2] Although these six stocks represent 0.5% of the average number of issues traded on a given day, they account for some 2.5% of all transactions during the period. However, the additional data reported in section 5 indicate that qualitatively the results apply to almost all traded issues.

Preliminary examination of a small segment of the entire sample suggests some interesting properties. In Figure 1, which contains data for Allied Chemical Corporation for the fourth day of the sample period, 29 of the 33 changes in price were less than 1/4 of a point away from the preceding transaction. This is consistent with Securities and Exchange Commission reports that 85% to 95% of all transactions in active stocks on the Exchange are less than 1/4 of a point removed from each other (15, p. 378).

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[1] Read $P(\Delta Y_t = X \mid \Delta Y_{t-1})$, as the probability that the change in Y_t equals X, given the change in Y_{t-1}.

[2] Because a complete record of transactions was not available for the American Tobacco Corporation, the seventh stock, we deleted it.

TABLE I: FREQUENCY TABLE OF CONSECUTIVE PAIRS OF PRICE CHANGES*

ΔY_{t-1}	ΔY_t							Totals
	−3/8	−2/8	−1/8	0/8	+1/8	+2/8	+3/8	
−3/8	0	0	3	9	3	4	2	21
−2/8	1	10	32	136	61	51	1	292
−1/8	0	35	231	1,059	777	80	3	2,185
0/8	4	130	1,128	3,139	1,041	130	3	5,575
+1/8	9	72	709	1,104	236	22	4	2,156
+2/8	5	48	64	129	40	6	1	293
+3/8	2	2	2	6	1	1	0	14
Totals	21	297	2,169	5,582	2,159	294	16	10,536

* Data compiled from the ticker quotations of the following stocks: Allied Chemical Corporation, Alcoa, American Can, A.T. & T., Anaconda, Bethlehem Steel, during the 22 trading days of the month of October, 1964.
Source: Francis Emory Fitch, Inc., Stock Sales on the New York Stock Exchange.

Though the number of transactions is small, Figure 1 suggests another phenomenon which has been mentioned in the literature, the "stickiness of even eighths." All sixteen of the zero changes occurred at the even eighths, even though there were three odd and two even eighth positions in the total sample.

Finally we observe a striking feature which pervades the entire sample of

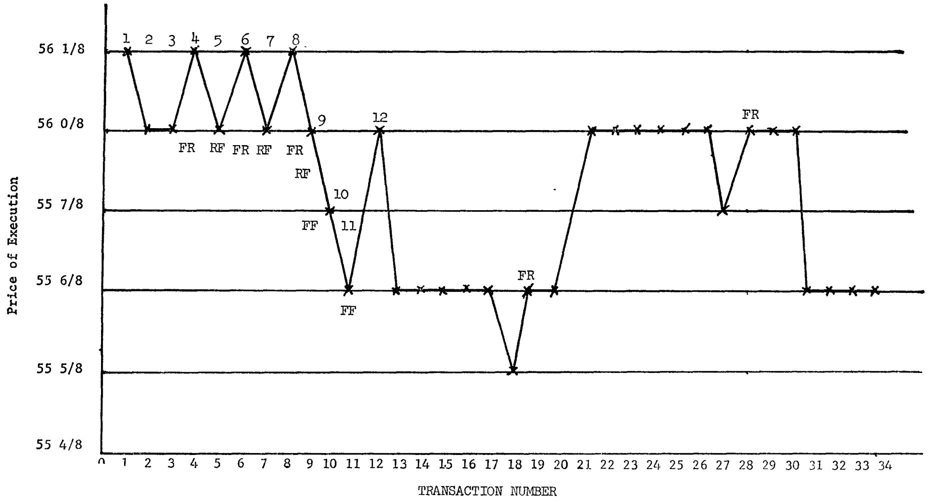

FIG. 1. Ticker transaction in Allied Chemical Corporation.* (For Day of October 6, 1964.)

* $F = \frac{1}{8}$ point fall, $R = \frac{1}{8}$ point rise. *FR* and *RF* are $\frac{1}{8}$ reversals. *FF* and *RR* are $\frac{1}{8}$ continuations.

TABLE II. TRANSITION MATRIX OF CONSECUTIVE PAIRS OF PRICE CHANGES

ΔY_{t-1}	ΔY_t							Marginal Probabilities
	−3/8	−2/8	−1/8	0/8	+1/8	+2/8	+3/8	
−3/8	.000	.000	.143	.429	.143	.190	.095	.002
−2/8	.003	.034	.110	.466	.209	.175	.003	.028
−1/8	.000	.016	.106	.485	.356	.037	.001	.207
0/8	.001	.023	.202	.563	.187	.023	.001	.529
+1/8	.004	.033	.329	.512	.109	.010	.002	.204
+2/8	.017	.164	.218	.440	.137	.020	.003	.027
+3/8	.143	.143	.143	.429	.071	.071	.000	.001
Marginal Probabilities	.002	.028	.206	.525	.205	.028	.001	1.0

price movements analyzed in this study. Most of the non-zero changes in price were opposite in direction to the preceding non-zero change: twelve in the opposite direction versus four in the same direction. When the signs of two non-zero consecutive changes are unlike each other, this pattern will be named a reversal, and when they are in the same direction, the pattern will be called a continuation. Thus, we have 12 reversals and four continuations in the price movements of Allied Chemical on October 6, 1964. Of these sixteen, only the 1/8 reversals and continuations are marked on Figure 1 (see section 5).

Considering the entire sample of transactions for six stocks during October 1964, we present the joint frequency distribution of consecutive pairs of changes in Table I and the estimated transition matrix derived from these changes in Table II. In row 5 of Table I, for example, the figure 2156 in the right margin is the total number of rises of 1/8 and the figure 709 is the number of these 2156 rises that were followed by a decline of 1/8. Thus, in Table II the ratio 709/2156 = 0.329 appears in row 5 indicating the fraction of all rises of 1/8 that were followed by a decline of 1/8. In standard notation,

$$P\left(\Delta Y_t = \frac{1}{8} \middle| \Delta Y_{t-1} = +\frac{1}{8}\right) = \frac{236}{2156} = 0.109, \quad \text{and}$$

$$P\left(\Delta Y_t = -\frac{1}{8} \middle| \Delta Y_{t-1} = \frac{1}{8}\right) = \frac{709}{2156} = 0.329. \tag{2}$$

The tendency for stock price movements to reverse direction shows up in Table II as negative correlation between ΔY_{t-1} and ΔY_t. Notice how the entries in the diagonal from lower left to upper right are all, except for the common one, larger than the corresponding entries in the diagonal from upper left to lower right. If the changes were truly independent—as assumed in a random walk model—both diagonals should be the same within the limits of random error. In addition, there should be no significant variation in the conditional distribution of ΔY_t over the tabulated values of ΔY_{t-1}. That is, all these conditional distributions should be the same as the marginal distribution, within the limits of random error.

A formal test for independence in transition matrices has been proposed by Anderson and Goodman [1]. Applied to Table II, this test has exactly the same form as a chi-square test for independence in a 7×7 contingency table. The chances of finding deviations from independence at least as large as those observed are approximated by $P(x^2 > 1147.9 | 36)$, an exceedingly small number. (The 99.99999999th percentile of the x^2 statistic with 36 d.f. is 106.) The variations in Table II cannot reasonably be attributed to chance.

To highlight this tendency toward reversal, we have abridged Table I by eliminating the no-change row and the no-change column and then consolidating the remaining entries into four classes; the result is a 2×2 table as follows:

	$\Delta Y_t < 0$	$\Delta Y_t > 0$	Total
$\Delta Y_{t-1} < 0$	312	982	1294
$\Delta Y_{t-1} > 0$	913	311	1224
Total	1225	1293	2518

It is apparent that two changes in opposite directions occur approximately three times as often as changes in the same direction.

A consequence of independence of successive price changes is that all subsets of price changes have the same frequency distribution. For example, the price changes following a change of $-3/8$ would have the same distribution (hence expected value) as the price changes after a $+3/8$ change. But this is not true. From row 1 of Table II we can see that after a change of $-3/8$, 14.3% of the next changes were declines of 1/8, 19.0% were advances of 2/8, and 9.5% were rises of 3/8. In other words, after a change of $-3/8$, the expected value of the next transaction is 0.67 eighths of a point, i.e., the sum of $(0.143)(-1/8) + (0.143)(1/8) + (0.190)(2/8) + (0.095)(3/8)$.

Similarly, we have calculated the average price change at transaction t corresponding to each of the other six changes at transaction $t-1$. These average changes are given below in eighths:

ΔY_{t-1}	$-3/8$	$-2/9$	$-1/8$	0/8	1/8	2/8	3/8
Average ΔY_t	0.67	0.38	0.30	-0.02	-0.37	-0.42	-0.64

As measured by a Kruskal-Wallis one-way analysis of variance, or otherwise, the tendency for the average to decrease as ΔY_{t-1} increases is obvious.

Finally, the data suggest that large changes tend to be followed by large changes. For example, 21.2% of the changes of 2/8 or more were followed by changes of 2/8 or more in absolute value, as were 22% of the changes of −2/8 or less. After moves of −1/8, 0.8, and 1/8, the percentages of subsequent changes which were at least 2/8 in absolute value were respectively 5.4%, 4.8% and 4.9%. These results may be exaggerated slightly by the possibility that a large change, followed by a large change in the opposite direction, may be a printing error on the ticker. But this eventuality is very unlikely because the degree of accuracy of the ticker is very high. For example, Leffler and Farwell report that on a day in which 30,000 transactions occur there is an average of only 10 printing errors on the ticker (7, p. 158).

3. SECOND ORDER EFFECTS IN STOCK PRICES

Turning now to the question of how satisfactory the first order Markov model is for describing the underlying process of price movements, we seek to determine the effect, if any, of ΔY_{t-2} on ΔY_t. To this end, we present the joint distribution of ΔY_{t-2}, ΔY_{t-1}, and ΔY_t in Table III. For simplicity in presentation and analysis, we have reduced the price movements to just five classes. We have combined into one class the changes of +2/8 and +3/8, and into another class the changes of −2/8 and −3/8. The arrangement of Table III is designed to permit analysis of the relation between ΔY_{t-2} and ΔY_t with ΔY_{t-1} held constant. Each row in each of the five tables shows the estimated probability distribution of ΔY_t for one combination of ΔY_{t-1} and ΔY_{t-2}. The right hand margin contains the total frequencies upon which the estimate is based.

For example, we learn from Table III-D that after a decline of 1/8 followed by a rise of 1/8, the relative frequency of rises of 1/8 on the next transaction is .108. Similarly, the entry in the second column and fourth row of Table III-E discloses that after a rise of 1/8 followed by a rise of +2/8 or greater, the relative frequency of declines of 1/8 was .190. The approximate significance levels are indicated at the bottom of each table.

Tables III A–III E consist of five 5 by 5 contingency tables. To test independence between ΔY_t and ΔY_{t-2}, Anderson and Goodman [1] propose that X^2 be calculated for each table, with the sum of the 5 X^2 values (with the appropriate degrees of freedom) serving as the test statistic for the null hypothesis.

By scrutinizing selected differences between proportions (or differences between differences between proportions), we can observe certain interesting properties of the price movements. For example, in Table III-C, a negative change followed by a change of 0/8 indicates that a rise on the next transaction is more likely than a decline, and a positive change followed by a zero change indicates a decline is more likely. That is

$$P(\Delta Y_t > 0 \mid \Delta Y_t \neq 0, \quad \Delta Y_{t-1} = 0, \quad \Delta Y_{t-2} = -\ 1/8) = 0.74$$
$$P(\Delta Y_t > 0 \mid \Delta Y_t \neq 0, \quad \Delta Y_{t-1} = 0, \quad \Delta Y_{t-2} = +\ 1/8) = 0.24. \qquad (3)$$

These probabilities are almost identical to the comparable first-order probabilities as derived from Table II. That is,

$$P(\Delta Y_t > 0 \mid \Delta Y_t \neq 0, \quad \Delta Y_{t-1} = -\ 1/8) = 0.76$$
$$P(\Delta Y_t > 0 \mid \Delta Y_t \neq 0, \quad \Delta Y_{t-1} = +\ 1/8) = 0.25. \qquad (4)$$

TABLE III. JOINT DISTRIBUTION OF PRICE CHANGES, ΔY_t, ΔY_{t-1}, AND ΔY_{t-2}*

Table	ΔY_{t-2}	ΔY_{t-1}	ΔY_t $\leq -2/8$	$-1/8$	$0/8$	$+1/8$	$\geq +2/8$	Total
III-A	$\leq -2/8$	$\leq -2/8$	.000	.000	.727	.182	.091	11
	$-1/8$	$\leq -2/8$	.061	.212	.333	.212	.182	33
	$0/8$	$\leq -2/8$	.039	.078	.437	.250	.195	128
	$+1/8$	$\leq -2/8$	.024	.106	.435	.200	.235	85
	$\geq +2/8$	$\leq -2/8$	.000	.151	.623	.094	.132	53
	Probability $P(X^2>15.8 \mid 9)<.0034$							
III-B	$\leq -2/8$	$-1/8$	.000	.235	.471	.118	.176	34
	$-1/8$	$-1/8$	.031	.159	.405	.308	.097	227
	$0/8$	$-1/8$	.016	.087	.496	.366	.035	1132
	$+1/8$	$-1/8$	.012	.101	.493	.381	.013	682
	$\geq +2/8$	$-1/8$	.031	.292	.338	.308	.031	65
	Probability $P(X^2 \geq 87.5 \mid 16)< 10^{-13}$							
III-C	$\leq -2/8$	$0/8$	.045	.127	.452	.223	.153	157
	$-1/8$	$0/8$	.008	.114	.527	.313	.038	1044
	$0/8$	$0/8$	.017	.194	.598	.173	.018	3128
	$+1/8$	$0/8$	.037	.317	.533	.106	.007	1085
	$\geq +2/8$	$0/8$	.208	.240	.408	.128	.016	125
	Probability $P(X^2 \geq 611.9 \mid 16)< 10^{-189}$							
III-D	$\leq -2/8$	$+1/8$	.047	.281	.484	.172	.016	64
	$-1/8$	$+1/8$	.018	.332	.538	.108	.004	766
	$0/8$	$+1/8$	.036	.342	.522	.088	.012	1024
	$+1/8$	$+1/8$	.073	.289	.440	.161	.037	218
	$\geq +2/8$	$+1/8$	.220	.171	.488	.098	.024	41
	Probability $P(X^2 \geq 39.7 \mid 16)<.001$							
III-E	$\leq -2/8$	$\geq +2/8$	.224	.155	.500	.103	.017	58
	$-1/8$	$\geq +2/8$	.200	.320	.400	.080	.000	75
	$0/8$	$\geq +2/8$	.120	.195	.481	.158	.045	153
	$+1/8$	$\geq +2/8$	.476	.190	.286	.000	.048	21
	$\geq +2/8$	$\geq +2/8$	.375	.125	.000	.500	.000	8
	Probability $P(X^2 \geq 4.0 \mid 4)<0.40$							

* Tables A and E were condensed due to paucity of data to 3×3 tables, giving (3−1)×(3−1)×(2)=8 degrees of freedom; whereas Tables B, C, and D were left as 5×5 tables, giving (5−1)×(5−1)×(3)=48 d.f. Thus, the sum of the five X^2 values for the table has a X^2 distribution with 56=48+8 d.f.

In this second-order pattern then, it appears that an issue behaved just as if the change of 0/8 had not occurred. In other words, the movement in price seems to be governed by the change which occurred before the zero change.

Results similar to this lead us to focus attention on continuations and reversals. The data of Table IIIA–IIIE may be incorporated into a 4×2 table by deleting the no-change row and no-change column and combining all positive changes and negative changes into two classes as follows.

ΔY_{t-2}	ΔY_{t-1}	$\Delta T_t<0$		$\Delta Y_t>0$		Total
<0	<0	60	.337	118	.663	178
$>$	<0	*117*	.256	*340*	.744	*457*
	Total	177	.279	458	.721	635
<0	>0	350	.759	111	.241	461
$>$	>0	*113*	.681	*53*	.319	*166*
	Total	463	.738	164	.262	627

(Absolute frequencies are given as integers and the transition probability estimates are given in parentheses.)

Notice that a negative change is 1.32 times as likely after two consecutive negative changes as after a positive change followed by a negative change (.337 vs .256). In addition, a positive change is 1.32 times as likely after two positive changes as after a negative change followed by a positive change (.319 vs .241).

Although these tables reemphasize the preponderant tendency for stock price movements to reverse direction, they indicate that the probability of reversal is not constant, but depends on the direction of previous movements. A reversal is more probable after a previous reversal than after a continuation; a continuation is more probable after a previous continuation than after a reversal.

An obvious next step in this analysis is to check whether an advance (decline) is more probable after three consecutive advances (declines) than after two advances (declines). For the six stocks in our Dow Jones sample, a continuation is approximately 1 1/2 times as likely after three consecutive continuations as after two continuations. Furthermore, after four continuations, a subsequent continuation is 1.27 times as likely as after three. Unfortunately, a paucity of data (only 65 occurrences of three consecutive continuations) prevents us from pursuing this line of analysis here.

These results came as a surprise to several readers who saw them in preliminary form.[3] In the next section, however, we hope to show that they are the natural consequence of the mechanics of trading on the stock exchanges.

4. THE MECHANICS OF COMPETITIVE MARKETS

The ability of customers to place orders at restricted prices as well as at current market prices is an essential feature of market making on the New York Stock Exchange, and on many similar markets. Approximately 60% of all executed orders on the NYSE are market orders. The most prevalent type of restricted order is labeled a limit order. Buy limits constrain the broker to execute the order at a specified price or lower, and conversely for sell limited orders. These orders are recorded on the book of the specialist, who receives

[3] Mr. C. Granger observed in a letter of June, 1965 that statistical methods based on the analysis of the autocovariance sequence of our data led him to the same conclusions as ours.

commissions for handling them. In addition to these commissions, which constitute riskless income, the specialist enjoys profits (and sometimes losses) by trading on his own account.

A customer's order to buy or sell at the market is transmitted to the appropriate broker on the floor of the Exchange. It is this floor broker's duty to obtain the best possible price available at the time. To do this, he goes to the post where the stock is traded and asks the specialist for a quote. Let us assume that the specialist is not trading for his own account. The specialist quotes his book by announcing the highest buy limit and lowest sell limit entered on it. As an illustration, a simulated page from an imaginary specialist's book appears in Table IV. The quote for the stock will be 33 4/8 bid, 33 5/8 asked. A market buy order would be executed at 33 5/8; a market sell at 33 4/8. The bid price differs from the asking price, and both exist concurrently in time.

There is no such thing as a single price at which stocks may actually be traded for time intervals as short as between consecutive transactions. The double-valued nature of potential executed prices (the quote) has important consequences for the sequence of actual executed prices.

Consider now what happens when a sequence of random buy and sell orders (without a preponderance of either), arrives at the post of the specialist whose book looks like Table IV. In the short run, the limit orders on the book will act as a barrier to continued price movement in either direction. Until all limit orders at the highest bid (33 4/8) and the lowest offer (33 5/8) are executed, transaction prices will fluctuate up and down between the bid and the offer in accordance with the random arrival of the market orders. Moreover, the period of oscillation may tend to last longer than a glance at the specialist's book would suggest; additional orders to buy at 33 4/8 and to sell at 33 5/8 are to be expected. Therefore, the pattern of numerous reversals displayed by the data exhibited in the previous sections is just what one might expect from the current system of trading on the Exchange.

Holbrook Working has reported a similar tendency to reversal in the intraday price movements of Chicago Wheat futures [19]. His sampling study included 143 series of 100 successive price changes covering the years 1927–1940. He reported that in 76 of the 143 series, the price changes of 1/8 of a cent in either direction were followed by opposite changes 75 or more times out of 100. Furthermore 140 of the price series considered contained 65 or more reversals per 100 changes.[4]

This tendency to reversal is to be expected in any market in which a broker controlling the supply of the commodity makes available a firm quote for a limited amount of time. Thus, suppose a coin dealer in uncirculated 1909-svdb pennies puts out a weekly quote sheet. A typical quotation might be \$265 bid, \$300 offered. For one week the price for all transactions (his and all others who read his quote sheet) will oscillate between those levels or at a slightly narrower spread if he has casual competitors. One can verify this by checking the transactions reported on the various teletypewriter systems, e.g., International Teletype Network.

[4] Only non-zero changes are reported by the Chicago Board of Trade.

TABLE IV. A PAGE FROM A SPECIALIST'S BOOK*

	Buy				Sell	
33	5 Smith 3 Abrot 2 Green 1 Jones 1 Elim 1 Lakis	3 Benton 15 Denoff 1 Fried	32	33		
1/8	1 Stahle 2 Vied		3	1/8		
2/8	2 James 1 Pratt 1 Gelb	4 Lurie	8	2/8		
3/8	1 Ford 1 Vernon		2	3/8		
4/8	2 Brown 1 White 7 Dell 1 Berger 1 Binder 1 Shoup	2 Gross 1 Hand	16	4/8		
5/8				5/8	1 Ross 1 Hunt	2
6/8				6/8	1 Lee 2 Block 2 Sims 1 Bloom	6
7/8				7/8	1 Dorf 1 Mann 1 Chan	3

* The simulated page from an imaginary specialist's book which is pictured here contains a record of the highest bids and the lowest offers. In this case it is 33 4/8 bid and 33 5/8 offered. For each order, the name of the broker giving the order, and the number of shares in the order, are entered in the proper location. It should be noted that the bid must be below the offers and offers above the bids, or else they would be executed immediately. This simulated book was constructed by adding the limit and stop orders at the eight fractional prices in each of the five books that have appeared in the literature.

If a sequence of market orders will generate reversals when buy orders and sell orders are equally numerous, it will generate continuations and runs when one type of order predominates. Suppose, for example, that the specialist of Table IV receives a sequence of market buy orders. The price of execution will be 33 5/8 when the first order comes in, but it will quickly advance to 33 6/8 as the limit orders at 33 5/8 are exhausted. With continued buy market orders, the price will soon reach 33 7/8. At this point, the statistical record will show a continuation—a rise from 33 5/8 to 33 6/8 followed by another rise from 33 6/8 to 33 7/8. And if more buy orders come in, the price will rise to 34, and the record will then show three successive rises—that is, one continuation followed by another.

Our analysis shows how the tape reader can infer the composition of market orders by observing the pattern of reversals and continuations in ticker prices. Frequent reversals suggest buy and sell orders in roughly equal proportions; more reversals are then to be expected. Absence of reversals suggests orders all on one side— to buy if price changes are up, to sell if price changes are down; a continuation of trend is then to be expected. A long sequence of transactions at one price suggests that market orders are all on one side but have not yet exhausted the limit orders at that one price. Were it not for the possibility of matching market orders, a long sequence of transactions at one price would seem to suggest that the next change in price would agree in direction with the last preceding change. But this conclusion has to be modified, since there is an unequal clustering of limit orders at different eighth positions.

The larger the number of limit orders at a given price level, the longer it will take a sequence of market buy orders to break through it. This increases the chances that just one market order to sell will come in, causing a reversal. For instance, in the case of the stock whose book is simulated in Table IV, we might expect with a preponderance of buy at market orders that the proportion of reversals after the sequence 33 4/8, 33 5/8 would be *less* than after the sequence 33 5/8, 33 6/8, because there are more limit orders (to sell) at 33 6/8 than at 33 5/8.

Further examination of Table IV reveals that the limit orders tend to cluster at the integer, half, quarters, and odd eighths in descending preference. This seems to be a prevailing characteristic of specialists' books (see legend, Table IV). We conclude from this discussion that reversals are more common at even eighths than at odd eighths, and more common at integers than half integers.

The mechanics of stock trading—as we have described them—are by no means peculiar to the New York Stock Exchange; they are closely matched on other American security exchanges, and they have their counterparts on the commodity exchanges. They may even have counterparts on other more or less organized competitive markets—say, stamps, coins, or used cars. These dealers must have both an inventory of cash with which to buy and an inventory of goods to sell. It may be instructive to compare the red and blue books of suggested buying and selling prices for used cars, and catalogues of market prices for U. S. stamps with the book of the specialist.

As reported above, the tendency to reversal has already been verified in coin and commodity markets. Numbers bookies frequently report clustering of numbers corresponding to some unusually symbolic event, and it is a commonplace thing for the $2 issue of a set of stamps to sell for more than the $5 issue (e.g., most U.S. and U.N. series). Thus, clustering at round numbers probably holds in many other markets. We are not familiar with any experimental data in other markets on relative chances of continuation after previous continuations and reversals.

There are numerous casual competitors of dealers in these secondhand markets. In addition, want ads provide further competition and information. There are only two stocks in which competing dealers operate on the N.Y.S.E., and none on the A.S.E. For this monopoly privilege the specialist is required to maintain a fair and orderly market by trading for his own account when necessary. It should not be assumed that these transactions undertaken by the specialist, and in which he is involved as buyer or seller in 24% of all market volume, are necessarily a burden to him. Typically, the specialist sells above his last purchase on 83% of all his sales, and buys below his last sale on 81% of all his purchases [15, p. 84]. An insight into his technique will be presented below.

Let us imagine that the price of the stock has had a rise during the day's trading. The specialist or a floor trader might take a short position at 7/8, knowing that a considerable excess of buy market orders over sell orders would be needed to push the price through the 8/8 level, there being an excessive number of limit orders at 8/8. At the worst, the specialist could take a 1/8 point loss by buying at the 8/8 value after all of the sell limit orders on his book have been filled. Conversely, by taking a long position at 1/8 after a decline to that level, the specialist would have a chance to profit by his participation.

The New York Stock Exchange reports that one of the specialist's functions is to stabilize the market in his stock. They test this by the stabilization or "tick test." All specialists' purchases below the last different price and sales above the last different price are considered stabilizing. The tendency to reversal and clustering of limit orders explains why such contra-tick trading should be profitable.

Mr. Alfred Cowles added the following observation in a letter of March, 1965. "If professionals actually do habitually profit from a knowledge of these patterns, that might explain a phenomenon which for many years has intrigued me. As a result of repeated analyses of large numbers of purchases and sales made through various brokers for investors' accounts, I have noted repeatedly that the average price at which series of 100 or more orders have been executed consistently averaged at prices slightly less favorable to the investors than the average of high and low for the day for each stock purchased or sold." This is a manifestation of the compensation the specialist receives for the stabilizing services he performs to investors. [See 13, p. 103.]

5. AN OBSERVATIONAL TEST OF PROPERTIES INDUCED BY MARKET MAKING

In this section we test a second sample of data for the properties suggested by the preceding discussion of market making. The data consist of the complete record of ticker transactions during a randomly chosen day in January of each of seven consecutive years. To reduce the magnitude of the computations to a manageable level, and to maintain a relatively homogeneous sample, we eliminated from consideration all transactions in which there was no change from the previous price. Quantitatively this reduced the number of transac-

TABLE V. RISES AND FALLS FOLLOWING THE FOUR EVENTS*

Terminating Fraction of Event	Event *RR*		Event *FR*		Event *FF*		Event *RF*		
	Number of Rises	Number of Falls	Number of Rises	Number of Falls	Number of Rises	Number of Falls	Number of Rises	Number of Falls	Total
0/8	132	169	150	476	171	73	433	140	1,764
1/8	94	137	138	422	165	66	428	132	1,682
2/8	104	130	146	419	141	85	418	178	1,621
3/8	108	153	157	390	119	63	348	145	1,483
4/8	150	157	130	326	139	69	338	147	1,456
5/8	116	135	138	316	168	85	409	144	1,511
6/8	108	146	167	406	141	81	434	176	1,659
7/8	103	173	155	463	104	80	503	140	1,721
Total	915	1,200	1,181	3,218	1,148	602	3,311	1,202	12,777

* Consideration of the way in which the four events FF, FR, RF, RR, and their subsequent moves are defined and taken from data will show that these 12,777 observations cannot be considered as independent, since each non-zero move appears once as the first member of an event RR, etc., once as the second member, and once as a subsequent move. As a lower limit there are then 12,777/3 strictly independent observations. To check whether this consideration might distort the results, we examined a synthetic random walk for the four events and random move, in the same way as the actual data was examined. No significant departures from equality in the equivalent eight columns of Table VI were found.

tions by almost 50%, but qualitatively the loss of information was small. We took this set of data as the sample and concentrated upon all price movements which followed two consecutive changes of $\pm 1/8$.

The sequences were classified in the following manner:

Event *RR*: A rise of $(+1/8)$ followed by a rise of $(+1/8)$.
Event *FR*: A fall of $(-1/8)$ followed by a rise of $(+1/8)$.
Event *FF*: A fall of $(-1/8)$ followed by a fall of $(-1/8)$.
Event *RF*: A rise of $(+1/8)$ followed by a fall of $(-1/8)$.

Examples of these events may be found in Figure 1. It may be observed that on the fourth transaction, Event *FR* (fall-rise) occurred at the fractional price 1/8, and a decline followed. On the fifth transaction, Event *RF* (rise-fall) occurred at the fractional price 0/8, and an advance followed.

This data sample is intended primarily to examine price structure at the different eighth positions. It is still reassuring that the total number of occurrences of the four events confirms the conclusions of our previous analysis. Recall that events *RF* and *FR* are reversals; events *RR* and *FF* are continuations. In the sample of 12,777 occurrences of these events, the ratio of events *FR* and *RF* to events *RR* and *FF* is $8912/3625 = 2.34/1$, a ratio quite compatible to the 3/1 ratio shown in Equations (3) and (4).

Additional confirmation of our previous conclusions can also be found in Table V and Figures 2 and 3. Table V contains the number of falls and rises (*not* restricted to $\pm 1/8$) which follow each of the four events for each of the eight fractional levels. The total number of rises and falls following all events terminating at a specific fractional price are given in the last column to the right. The total at the bottom of each column represents the total number of

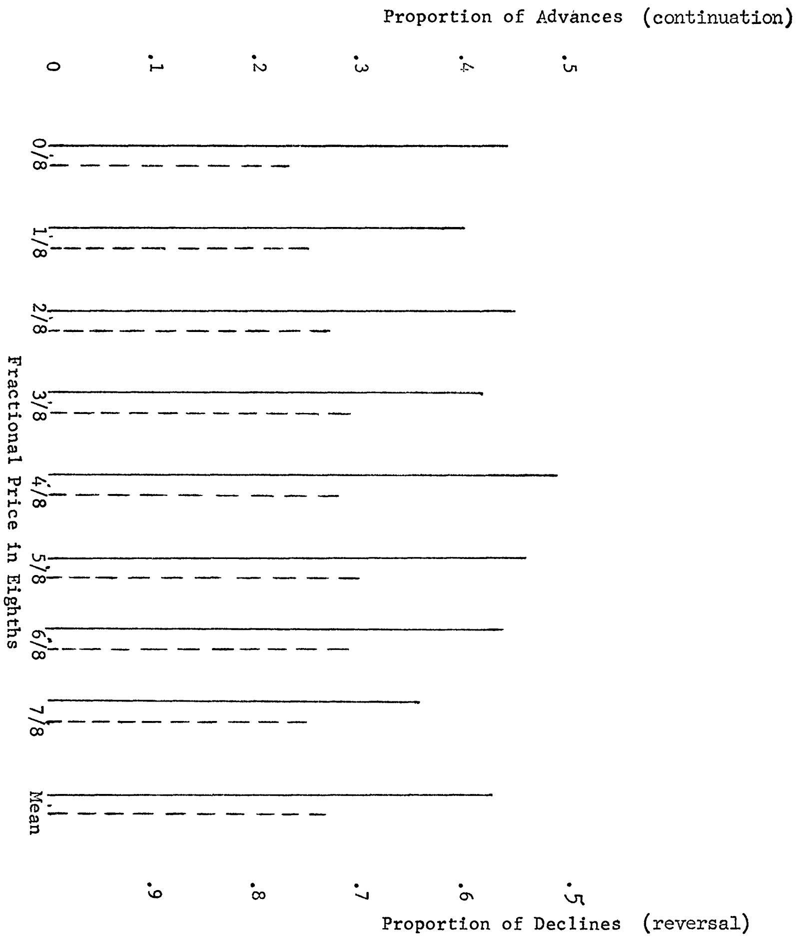

FIG. 2. Proportion of advances following events *RR* and *FR*.*

* Solid line shows proportion of advances after pattern RR. Dotted line shows proportion of advances after pattern FR. The coefficients of variation for the proportions represented by solid lines and dotted lines are about 7%.

rises or falls for all fractional prices after a specific event. Figure 2 contains the derived probability of a rise (1/8 or more) after patterns, or Events *RR* and *FR*. Figure 3 is a graph of the probability of a fall (1/8 or more) after Events *FF* and *RF*. The solid line in both figures indicates the probability of a continuation after two moves in the same direction. The right hand ordinate scale gives the probability of reversal.

The dotted line gives the probability of a continuation in the direction of the last move after two moves in an opposite direction (*FR*, *R* for Figure 2; *RF*, *F* for Figure 3). The last pair of lines to the right of the Figures gives the average probabilities for all the fractional price movements after the event. These were derived from the column marginals in Table V. For example, in the bottom row of Table V under Event *FF* there were 602 falls and 1148 rises. The probability of continuation was $602/1750=0.35$. The last solid line at the right of Figure 3 rises to this mark on the scale.

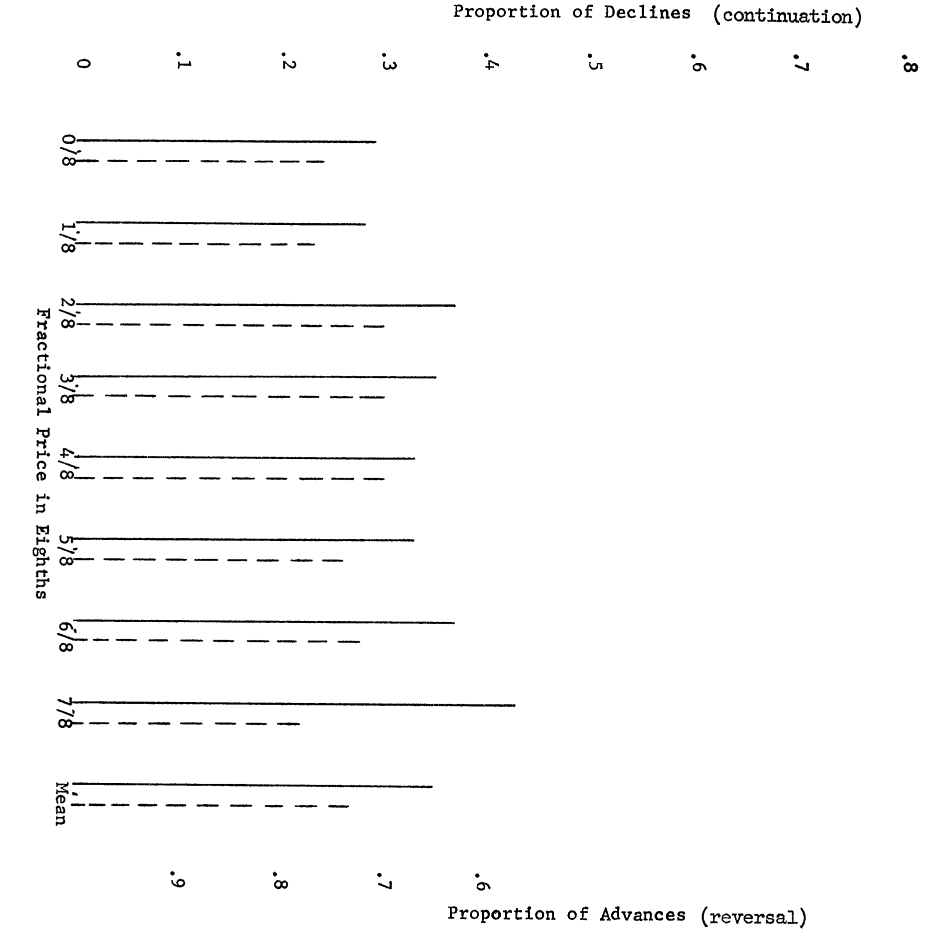

FIG. 3. Proportion of declines following events FF and RF.*

* Solid lines show proportion of declines after pattern FF. Dotted lines show proportion of declines after pattern RF. The coefficients of variation for the proportions represented by both the dotted and the solid lines are about 7%.

It is apparent that the solid line is taller than the dotted line at all positions in Figures 2 and 3. This indicates that continuations are more likely after two changes of the same sign than after two changes in opposite directions. But note that even after two changes in the same direction, reversals are still more probable than continuations at all eighths. Although these results come from a separate sample, they are in accord with the information displayed in section 2. A tendency to reversal is a property of market making, and the consistent difference between full and dotted lines of Figures 2 and 3 imply that continuations are slightly more probable after a previous continuation, than after a previous reversal (i.e., ΔY_{t-2} does influence ΔY_t).

It is noteworthy that the differences between the size of the solid and dotted lines at each fractional price is systematically greater for Figure 2 than for Figure 3. This shows that for this sample, rises at $t-2$ have a more pronounced effect (in the probability sense) on ΔY_t than do falls. Somehow this runs counter to our intuitive picture of the market in its stochastic structure as up and down symmetric. The result may arise from a slight preponderance of advances between consecutive transactions in the sample.

Before turning to the specific predictions, we repeat that a continuation or reversal of a move after events *RR, RF, FR, FF* is determined by agreement or disagreement with the sign of the terminating move of the event. Thus a continuation after event *FR* occurs when a fall of 1/8 followed by a rise of 1/8 is followed by a rise (of 1/8 *or more*); a reversal occurs if the third move is a fall. The notation P_c (7/8) refers to the probability of continuation after one of the four events ending at fractional price 7/8.

We come now to the specific predictions relating to fractional moves in prices. There are two facts from which our predictions are derived:

1. Limit orders tend to cluster more strongly at 8/8 than 4/8, and less strongly elsewhere. One expects, for example, to find continuations less likely after any of the four events ending at fractional price 8/8 than at events ending at 2/8.
2. Since the limit orders act as a barrier to continued price movement, the specialist and his floor trading competitors have a special incentive to sell for their own accounts one eighth below 8/8 and 4/8 after Events *RR* and *FR*, and to purchase one eighth above 8/8 and 4/8 after Events *RF* and *FF*. The incentive should be strongest at prices 1/8 away from the fractional price 8/8. For example, we predict that the relative frequency of continuation should be less after occurrences of Event *FF* terminating at fractional price 1/8 than after an occurrence of *FF* ending at price 5/8.

Forty predictions derived from these items are set forth in Table VI. The predicted inequality between chances of continuation at two different fractional prices are given in columns 1 and 4. The numbers under the four events in columns 2, 3, 5, and 6 refer to the actual difference between chances of continuation as calculated from Figure 2 and Figure 3. For example, the first predicted inequality in Table VI is that, when an event terminates at 0/8, the chances of a continuation are less than when an event terminates at 4/8. That is

$$P_c(0/8) - P_c(4/8) < 0.$$

The corresponding number under Event *RR* in column 2 shows that for the specific Event *RR*, the difference

$$P_c(0/8) - P_c(4/8) = -0.05.$$

All these occasions in which the facts were not in agreement with theory are underlined.

Assuming somewhat heroically that the chances of a correct prediction were 1/2 and independent of the success of any other prediction, we find that the binomial probability of 29 or more successes (the observed number in Table VI) out of 40 trials is a mere 0.003. In addition, examination of the table will disclose that the observed size of the difference between proportions of continuations was greater for the correct predictions than for incorrect predictions.

Note that the chance of a continuation after Event *RR* terminating at price

TABLE VI. PREDICTED AND OBSERVED DIFFERENCES BETWEEN CHANCES OF CONTINUATION

Figure (2)	Observed Value of Left-hand Side of Inequality in (1)		Figure (3)	Observed Value of Left-hand Side of Inequality in (4)	
Predicted Inequality	For Event *RR*	For Event *FR*	Predicted Inequality	For Event *FF*	For Event *RF*
(1)	(2)	(3)	(4)	(5)	(6)
$P_c(0/8)-P_c(4/8)<0$	−.05	−.05	$P_c(0/8)-P_c(4/8)<0$	−.04	−.06
$P_c(0/8)-P_c(2/8)<0$	−.01	−.05	$P_c(0/8)-P_c(2/8)<0$	−.10	−.06
$P_c(0/8)-P_c(6/8)<0$	+.01	−.06	$P_c(0/8)-P_c(6/8)<0$	−.08	−.04
$P_c(4/8)-P_c(2/8)<0$	+.04	+.00	$P_c(4/8)-P_c(2/8)<0$	−.06	+.00
$P_c(4/8)-P_c(6/8)<0$	+.06	−.01	$P_c(4/8)-P_c(6/8)<0$	−.04	+.02
$P_c(7/8)-P_c(5/8)<0$	−.10	−.04	$P_c(5/8)-P_c(7/8)<0$	−.09	+.04
$P_c(7/8)-P_c(3/8)<0$	−.06	−.03	$P_c(1/8)-P_c(5/8)<0$	−.06	−.02
$P_c(7/8)-P_c(1/8)<0$	−.04	+.00	$P_c(1/8)-P_c(7/8)<0$	−.15	+.02
$P_c(3/8)-P_c(5/8)<0$	−.04	−.01	$P_c(5/8)-P_c(3/8)<0$	−.01	−.04
$P_c(3/8)-P_c(1/8)<0$	+.02	+.03	$P_c(1/8)-P_c(3/8)<0$	−.07	−.06

4/8 appears to be out of line with the chance of a continuation after an occurrence of this event at any other fractional price. An explanation was offered by an odd lot broker on the New York Stock Exchange. He suggested that since limit orders cluster at fractional price 4/8 and 8/8, single transactions involving large volume would probably be traded at these levels. But the transactions of 1000 or more shares are printed out in full on the tape. That is, 175 *T* 59 1/2 as against *T* 58 5/8 for small orders. And "everbody knows that tape readers will rush in on the *same side*[5] of the market as the large orders, thus continuing the move. If this is true, continuations in price ought to be more likely when the preceding transaction was of 1000 or more shares.

Some interesting properties of price movement were masked by our technique. We considered movements of 1/8 or more only after the occurrence of two consecutive non-zero changes of 1/8. Therefore, the total number of rises and falls at the even and odd eights listed in Table V does not provide a reasonable estimate of the probability that a stock transaction took place at an even eighth. In fact, the last different price from the terminal price of all the events in our sample at even (odd) eighths must have occurred at odd (even) eighths. Thus the distribution of terminal fractional prices of events *RR*, *RF*, *FR*, and *FF* is biased to make the numbers of transactions at even and odd eighths nearly equal. A separate investigation, reported elsewhere [11], gives more complete information which bears on the relative frequency of odd and even eighths. An examination of all transactions on the NYSE in 1964 showed that 58.5% of all transactions on the NYSE during 1964 fell on an even eighth. The symmetric 95% confidence limits were at 55.9% and 61.1%. This preference

[5] By "same side" we mean that everybody rushes in to buy on reading 175 T 59 1/2 following a previous lower price.

for even eighths is largely a consequence of the tendency for 62% of all transactions at the same price as the previous price to occur at even eighths. This, in turn, is a consequence of the heavy concentration of limit orders at even eighths (six to one in the typical specialist's book of Table IV).

6. CONCLUDING REMARKS

The record of stock market ticker transactions displays four nonrandom properties: (1) There is a general tendency for price reversal between trades. (2) Reversals are relatively more concentrated at integers where stable slow-moving participants offer to buy and sell. There is a concentration of particular types of reversals just above and below these barriers. (3) Quick moving competitors cognizant of these barriers can take positions at nearby prices, thus "getting the trade" and hoping to make a profit. (4) After two changes in the same direction, the chances of continuation in that direction are greater than after changes in opposite directions.

It would be interesting to see if these properties of stock market prices hold in other markets. We remarked that the tendency to reversal holds in wheat and coin markets. As far as we know, no one has provided information concerning properties (2)–(4) in other markets.

Although the specific properties reported in this study have a significance from a statistical point of view, the reader may well ask whether or not they are helpful in a practical sense. Certain trading rules emerge as a result of our analysis. One is that limit and stop orders should be placed at odd eighths, preferably at 7/8 for sell orders and at 1/8 for buy orders. Another is to buy when a stock advances through a barrier, and to sell when it sinks through a barrier. Professional traders will recognize these rules or their equivalent as quite familiar.[6] Since the tendency of traders to prefer integers seems to be a fundamental and stable principle of stock market psychology, we may have confidence that the transactions of those who follow the proposed rules will not destroy the effect [3, 20].

Godfrey and his co-workers, have looked for periodicities and other regularities in the record of ticker transactions of 2 NYSE issues. Their conclusions are opposite to ours in a great many respects. The interested reader is invited to form his own conclusion by perusal of the references [refs. 5, and 8–14]. We shall be content here to record our impression that spectral analysis, the technique they utilized, seems unsuited to the analysis of stock market prices.

At a more fundamental level, the present writers believe that the discoveries of regularities in price movements of consecutive transactions reported herein provide a stepping stone for further and more exhaustive studies. The first step in this direction would be to derive the probability density function for daily stock price changes by letting the second order Markov process we have described run for the actual number of transactions that occur in different stocks during the day. Will the distribution of daily price changes approach normality? Will it *be* dependent on previous daily price changes and volume?

[6] The off-floor trader who uses a limit order does not know who betters his bid or offer, nor *exactly* when his order is executed. He is at a disadvantage relative to traders on the floor, since the on-floor trader knows, at this instant, the off-floor trader's bid and offer, but not conversely.

What is the best way to incorporate any existing dependence between price and volume movements into this process? Somehow one must incorporate both a "transaction number time scale," and a "calendar time scale" into the process, since there is evidence that both are significant [see, e.g., ref. 12, Fig. 9].

One fruitful approach might be to apply central limit theorems for dependent variables to the sum of price changes differenced over a constant number of transactions. The distribution of this sum, for $n>30$ is probably very close to normal. But daily price changes may be the sum of widely differing numbers of transactions. Perhaps daily price changes can be envisioned as a mixture of normal processes with weights proportional to observed classes of transaction numbers.

It is our hope that this paper will suggest questions and tests of this kind, and also help to solve them. Certainly the findings of structure, regularities, and dependence effects, which have been the subject of this study, ought to be valuable guides in the formulation of more sophisticated models of stock price movement.

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